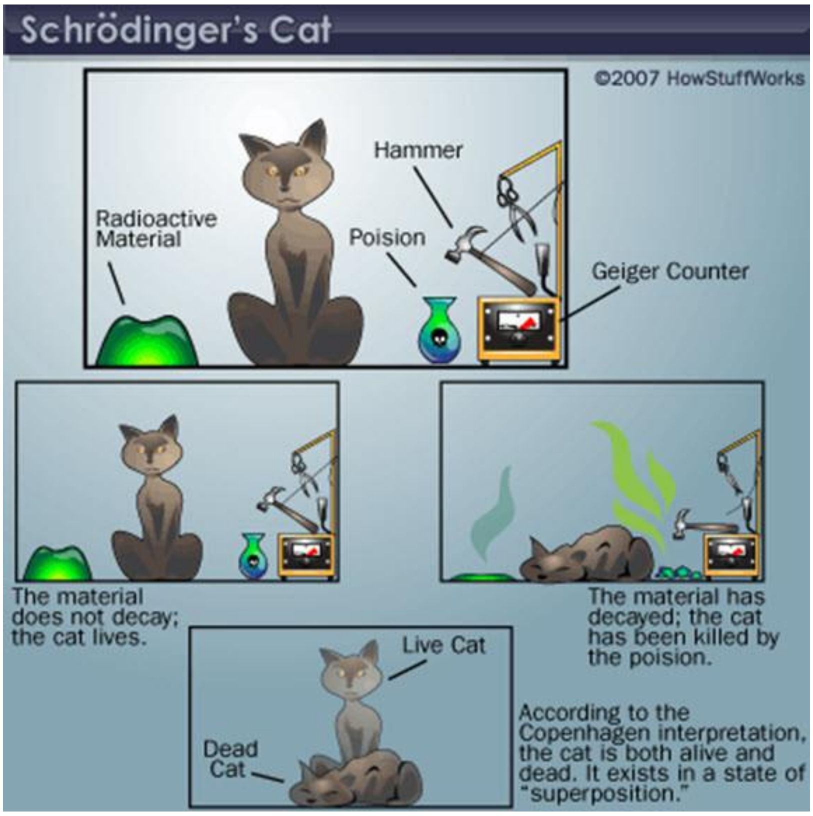
Photonics: Project 2A (2021)

Spooky action at a distance?

« Notions of causality and of the impossibility of being at several places at the same time are shattered by the quantum theory. The idea of superposition – of "being at two places at once" – is related to the phenomenon of entanglement. But entanglement is even more dramatic, for it breaks down the notion that there is meaning to spatial separation. Entanglement can be described as a superposition principle involving two or more particles. Entanglement is a superposition of the states of two or more particles, taken as one system. Spatial separation as we know it seems to evaporate with respect to such a system. Two particles that can be miles, or light years, apart may behave in a concerted way: what happens to one of them happens to the other one instantaneously, regardless of the distance between them. »

Amir Dan Aczel (1950 – 2015) (New York: *Entanglement – The Greatest Mystery in Physics*, Wiley, 2003, page 250)



« Much to the relief of cat-lovers, there is no need to perform the Scrödinger cat experiment in the laboratory. Paradoxes of this type are not found in the macroscopic world, because large systems consisting of many particles lose their quantum coherence through interactions with the noisy macroscopic environment. »

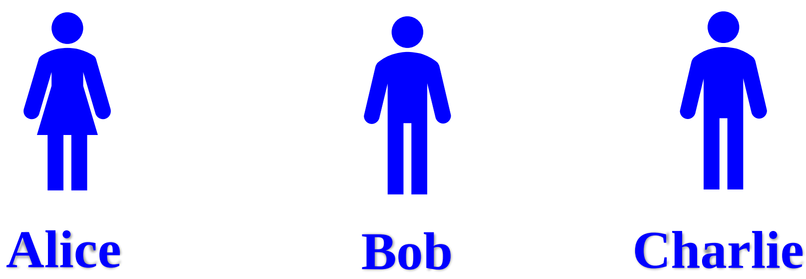
Anthony Mark Fox (Oxford, UK: *Quantum Optics – An Introduction*, Oxford University Press, 2006, page 298)

« Bell inequalities describe violations of local realism on average, meaning that each individual experimental outcome could occur in a local realistic theory. It is the statistics of these outcomes that violates local realism. In contrast, another class of quantum states, so-called Greenberger-Horne-Zeilinger (GHZ) states, which contain three particles, can be shown to violate local realism in each measurement outcome. »

Jonathan A. Jones and Dieter Jaksch (Cambridge, UK: *Quantum Information, Computation and Communication*, Cambridge University Press, 2012, page 163)

The main goal of this project is to confront the *local realism* of EPR (Einstein-Podolsky--Rosen) with the *quantum nonlocality* which the so-called GHZ puzzle puts in evidence and where three particles are sent to three different players.

In this GHZ game we have three players: Alice, Bob and Charlie.



All players are on the same team. The game consists of many rounds. In each round the players will be separated and receive a particle entangled with the other two. Then, each player will also receive a query (*input*) and will have to provide an answer (*output*). There is a referee (or *verifier*) who distributes the inputs and collects the outputs.

$$\textbf{Alice} \;\to\; \left\| \begin{array}{lll} \textbf{input} & \to & \boxed{x \in \{0,1\}} \\ \textbf{output} & \to & \boxed{a_x \in \{-1,+1\}} \end{array} \right.$$

$$\textbf{Bob} \;\to\; \left\| \begin{array}{lll} \textbf{input} & \to & \boxed{y \in \{0,1\}} \\ \textbf{output} & \to & \boxed{b_y \in \{-1,+1\}} \end{array} \right.$$

$$\textbf{Charlie} \;\to\; \left\| \begin{array}{lll} \textbf{input} & \to & \boxed{z \in \{0,1\}} \\ \textbf{output} & \to & \boxed{c_z \in \{-1,+1\}} \end{array} \right.$$

So, for each round (and for the three distributed entangled particles) we have pre-established the global output $(a_0, a_1;\ b_0, b_1;\ c_0, c_1)$ – according to the EPR interpretation (*local realism*). From these six numbers we get a well-defined value for the derived quantity

$$m = a_0 b_0 c_1 + a_0 b_1 c_0 + a_1 b_0 c_0 - a_1 b_1 c_1 .$$

Now, either $c_0 = c_1$ or $c_0 \neq c_1$ (in which case we must have $c_0 = -c_1$). Hence,

$$\left\| \begin{array}{lll} c_1 = c_0 & \Rightarrow & m = m' = a_0 c_0 (b_0 + b_1) + a_1 c_0 (b_0 - b_1) \\ c_1 = -c_0 & \Rightarrow & m = m'' = a_1 c_0 (b_0 + b_1) - a_0 c_0 (b_0 - b_1) \end{array} \right.$$

$$\left[\!\left[\begin{array}{lll} b_1 = b_0 & \Rightarrow & m' = a_0\, c_0\,(b_0 + b_1) = 2\, a_0\, b_0\, c_0 \in \{-2, +2\} \\ b_1 = -\, b_0 & \Rightarrow & m'' = -\, a_0\, c_0\,(b_0 - b_1) = -\, 2\, a_0\, b_0\, c_0 \in \{-2, +2\} \end{array}\right.\right.$$

Accordingly, one gets:

$$\boxed{m = +\, 2 \quad \text{or} \quad m = -\, 2}\,.$$

Since the average of a sum is the sum of the averages, we obtain

$$\mathbb{M} = \langle m \rangle ,$$

where we have introduced the **score** of this GHZ puzzle as

$$\boxed{\mathbb{M} = \langle a_0\, b_0\, c_1 \rangle + \langle a_0\, b_1\, c_0 \rangle + \langle a_1\, b_0\, c_0 \rangle - \langle a_1\, b_1\, c_1 \rangle}\,.$$

We are defining

$$\boxed{\mathbb{E}_{xyz} = \left\langle\, a_x\, b_y\, c_z \,\right\rangle} \quad \Rightarrow \quad \boxed{\mathbb{M} = \mathbb{E}_{001} + \mathbb{E}_{010} + \mathbb{E}_{100} - \mathbb{E}_{111}}\,.$$

As usual we represent the orthonormal basis of the three-dimensional Euclidean space as follows

$$\mathscr{B}\left(\mathbb{R}^3\right)=\{\mathbf{e}_1,\mathbf{e}_2,\mathbf{e}_3\},\quad \mathbf{e}_3=\mathbf{e}_1\times\mathbf{e}_2,\quad \mathbf{e}_i\cdot\mathbf{e}_j=\delta_{ij}=\begin{cases}1, & i=j\\ 0, & i\neq j\end{cases}$$

and then we make the following identifications

$$\boxed{\begin{array}{l} a_0=b_0=c_0=\mathbf{e}_2\\ a_1=b_1=c_1=-\,\mathbf{e}_1\end{array}}\,.$$

Accordingly, we have

$$\boxed{\begin{array}{lllll} \mathbb{E}_{001}=\langle a_0\,b_0\,c_1\rangle & \mapsto & -\,\mathbb{E}\left(\mathsf{Y}_1,\mathsf{Y}_2,\mathsf{X}_3\right) & \mapsto & -\left\langle\sigma_y\otimes\sigma_y\otimes\sigma_x\right\rangle\\ \mathbb{E}_{010}=\langle a_0\,b_1\,c_0\rangle & \mapsto & -\,\mathbb{E}\left(\mathsf{Y}_1,\mathsf{X}_2,\mathsf{Y}_3\right) & \mapsto & -\left\langle\sigma_y\otimes\sigma_x\otimes\sigma_y\right\rangle\\ \mathbb{E}_{100}=\langle a_1\,b_0\,c_0\rangle & \mapsto & -\,\mathbb{E}\left(\mathsf{X}_1,\mathsf{Y}_2,\mathsf{Y}_3\right) & \mapsto & -\left\langle\sigma_x\otimes\sigma_y\otimes\sigma_y\right\rangle\\ \mathbb{E}_{111}=\langle a_1\,b_1\,c_1\rangle & \mapsto & -\,\mathbb{E}\left(\mathsf{X}_1,\mathsf{X}_2,\mathsf{X}_3\right) & \mapsto & -\left\langle\sigma_x\otimes\sigma_x\otimes\sigma_x\right\rangle \end{array}}\,.$$

The letter $\mathbb{M}$ is a homage to N. David Mermin. So, after a large number N of runs, we always observe the following Mermin inequality:

$$\mathsf{EPR}\quad\mapsto\quad\boxed{-2\le\mathbb{M}_{\mathsf{EPR}}\le 2}\,.$$

Notice that this is the result we should obtain **if** our world would behave in terms of **local realism**, i.e., according to EPR.

However, according to quantum physics (or, using the more common denomination of QM as in *quantum mechanics*), we are measuring the entangled GHZ state of **three entangled particles** defined as

$$\textbf{GHZ entangled state} \quad \to \quad \boxed{\left|\mathrm{GHZ}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|000\right\rangle + \left|111\right\rangle\right)}.$$

One has

$$\left|\mathrm{GHZ}\right\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\0\\0\\0\\0\\1\end{pmatrix} \quad \Rightarrow \quad \left\langle \mathrm{GHZ}\middle|\mathrm{GHZ}\right\rangle = \frac{1}{2}\begin{pmatrix}1 & 0 & 0 & 0 & 0 & 0 & 0 & 1\end{pmatrix}\begin{pmatrix}1\\0\\0\\0\\0\\0\\0\\1\end{pmatrix} = 1.$$

For example, we may write

$$\left\langle \sigma_y \otimes \sigma_x \otimes \sigma_y \right\rangle = \left\langle \mathrm{GHZ}\right|\left(\sigma_y \otimes \sigma_x \otimes \sigma_y\right)\left|\mathrm{GHZ}\right\rangle.$$

In fact, using the mathematical formalism of quantum mechanics, we do get (you should prove this result)

$$\mathrm{QM} \quad \mapsto \quad \boxed{\begin{array}{l} \mathbb{E}_{001} = \left\langle a_0\, b_0\, c_1 \right\rangle = +1 \\ \mathbb{E}_{010} = \left\langle a_0\, b_1\, c_0 \right\rangle = +1 \\ \mathbb{E}_{100} = \left\langle a_1\, b_0\, c_0 \right\rangle = +1 \end{array}}$$

alongside with (you should also prove this result)

$$\text{QM} \quad \mapsto \quad \boxed{\mathbb{E}_{111} = \langle a_1 b_1 c_1 \rangle = -1}.$$

Therefore, according to quantum mechanics, we should always get (assuming perfect correlations or anti-correlations)

$$\text{QM} \quad \mapsto \quad \boxed{\mathbb{M}_{\text{QM}} = 4}.$$

Conclusion: Using the three-particle GHZ entanglement, a full disagreement between **local realism** (advocated by EPR) and the weirdness of **quantum mechanics** is revealed. Nevertheless, all experimental evidence corroborates that our world is governed by the laws of quantum mechanics.

In the essay herein proposed it is also expected from you to prove the incompatibility between the rule

$$\text{QM} \quad \mapsto \quad \boxed{\mathbb{E}_{111} = \langle a_1 b_1 c_1 \rangle = -1}$$

and what is expected according to EPR. In fact, according to EPR, we should get, instead,

$$\text{EPR} \quad \mapsto \quad \boxed{\mathbb{E}_{111} = \langle a_1 b_1 c_1 \rangle = +1},$$

after assuming, as our starting point, that

$$\text{QM+EPR} \quad \mapsto \quad \boxed{\begin{array}{l} \mathbb{E}_{001} = \langle a_0 b_0 c_1 \rangle = +1 \\ \mathbb{E}_{010} = \langle a_0 b_1 c_0 \rangle = +1 \\ \mathbb{E}_{100} = \langle a_1 b_0 c_0 \rangle = +1 \end{array}}.$$

Accordingly, we should get (in these ideal circumstances)

$$\text{EPR} \quad \mapsto \quad \boxed{\mathbb{M}_{\text{EPR}} = 2}.$$

« The best explanation anybody has come up with to this day is to insist that no explanation is needed beyond what one can infer from the laws of quantum mechanics. Those laws are correct. Quantum mechanics works. There is no controversy about that. What fail to work are attempts to provide underlying mechanisms, that go beyond the quantum-mechanical rules, for how certain strong quantum correlations can actually operate. One gets puzzled only if one tries to understand how the rules can work not only for the actual situation in which they are applied, but also in alternative situations that might have been chosen but were not. »

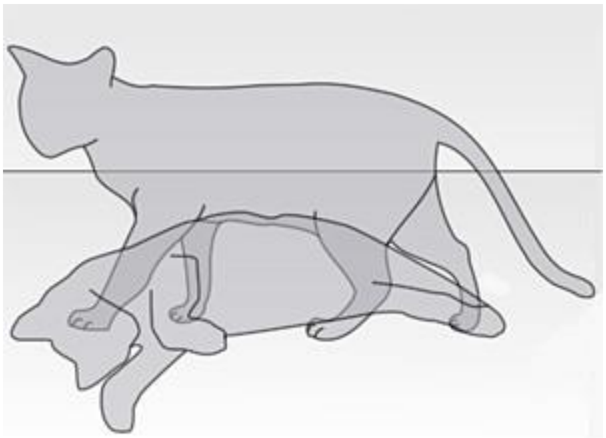
N. David Mermin (Cambridge, UK: *Quantum Computer Science – An Introduction*, Cambridge University Press, 2007, pp. 157 – 158)

« *Bell nonlocality provides the most compelling certification of the correct functioning of some quantum devices*, like those required to perform quantum cryptography and quantum computation. »

Valerio Scarani (Oxford, UK: *Bell Nonlocality*, Oxford University Press, 2019, page 4)

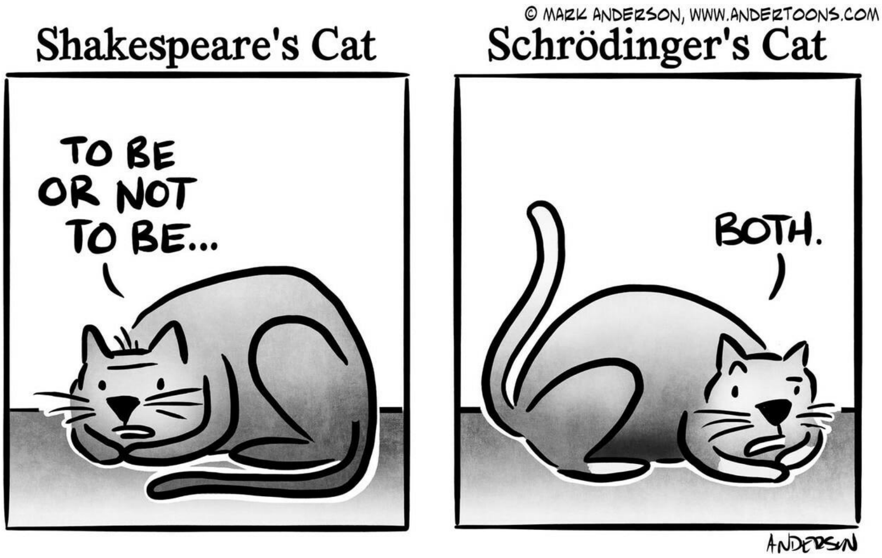
« In a classical world, we would expect that measurement outcomes are independent of the measurement process, and the results obtained at one location are independent of any actions performed at distances where information cannot be exchanged even at the speed of light. Recent experiments show that quantum mechanics does properly predict the results of experiments that violate the EPR criteria of reality and locality. »

Malin Premaratne and Govind P. Agrawal (Cambridge, UK: *Theoretical Foundations of Nanoscale Quantum Devices,* Cambridge University Press, 2021, page 55)



« If the things we are observing are so minute and delicate, and if the processes available for us to observe them are unavoidably clumsy and disruptive, then there may indeed be no legitimate way to abstract from an act of observation something we can associate only with the particle we observe, independently of how we choose to observe it. »

N. David Mermin (Cambridge, UK: *Boojums All the Way Through – Communicating Science in a Prosaic Age*, Cambridge University Press, 1990, page 119)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Shakespeare's Cat
TO BE OR NOT TO BE...
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BOTH.
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"About your cat, Mr. Schrödinger—I have good news and bad news."